Package 'mlogit'

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Title multinomial logit model

Author Yves Croissant <yves.croissant@univ-reunion.fr>

Maintainer Yves Croissant <yves.croissant@univ-reunion.fr>

Depends R (>= 2.10), Formula, statmod, lmtest, maxLik, zoo, MASS

Suggests car, nnet, AER, lattice

Description Estimation of the multinomial logit model

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Repository CRAN

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R topics documented:

`Beef` *Choice of a beef product*

Description

panel data

number of observations : 5 scenario for 47 individuals

observation : individuals

country : United States

Usage

```
data(Beef)
```

Format

A dataframe containing :

id individuals identifier

product one of `fresh`, `lean`, `dietLean`, `organic` and `noPurchase`

scenario scenario identifier

choice one if the product is chosen, 0 otherwise

price the price of the product

Source

Jae Bong Chang and Jayson L. Lusk (2010) "Mixed logit models: accuracy and software choice", *Journal of Applied Econometrics*.

Chang, J.B., J.L. Lusk, and F.B. Norwood (2009) "How Closely Do Hypothetical Surveys and Laboratory Experiments Predict Field Behavior?" *American Journal of Agricultural Economics*, **91**, pp. 518-34.

References

Journal of Applied Econometrics data archive : http://jae.wiley.com/jae/.

`Car` *Stated Preferences for Car Choice*

Description

a cross-section

number of observations : 4654

observation : individuals

country : United States

Usage

```
data(Car)
```

Format

A dataframe containing :

choice choice of a vehicule amoung 6 propositions

college college education ?

hsg2 size of household greater than 2 ?

coml5 commulte lower than 5 miles a day ?

typez body type, one of regcar (regular car), sportuv (sport utility vehicule), sportcar, stwagon (station wagon), truck, van, for each proposition z from 1 to 6

fuelz fuel for proposition z, one of gasoline, methanol, cng (compressed natural gas), electric.

pricez price of vehicule divided by the logarithme of income

rangez hundreds of miles vehicule can travel between refuelings/rechargings

accz acceleration, tens of seconds required to reach 30 mph from stop

speedz highest attainable speed in hundreds of mph

pollutionz tailpipe emissions as fraction of those for new gas vehicule

sizez 0 for a mini, 1 for a subcompact, 2 for a compact and 3 for a mid–size or large vehicule

spacez fraction of luggage space in comparable new gas vehicule

costz cost per mile of travel (tens of cents) : home recharging for electric vehicule, station refueling otherwise

stationz fraction of stations that can refuel/recharge vehicule

Source

McFadden, Daniel and Kenneth Train (2000) "Mixed MNL models for discrete response", *Journal of Applied Econometrics*, **15(5)**, 447–470.

References

Journal of Applied Econometrics data archive : http://jae.wiley.com/jae/.

`Catsup` *Choice of Brand for Catsup*

Description

a cross-section

number of observations : 2798

observation : individuals

country : United States

Usage

```
data(Catsup)
```

Format

A dataframe containing :

id individuals identifiers

choice one of heinz41, heinz32, heinz28, hunts32

disp.z is there a display for brand z ?

feat.z is there a newspaper feature advertisement for brand z ?

price.z price of brand z

Source

Jain, Dipak C., Naufel J. Vilcassim and Pradeep K. Chintagunta (1994) "A random–coefficients logit brand–choice model applied to panel data", *Journal of Business and Economics Statistics*, **12(3)**, 317.

References

Journal of Business Economics and Statistics web site : http://www.amstat.org/publications/jbes/.

correlation *Correlation structure of the random parameters*

Description

Functions that extract the correlation structure of a mlogit object

Usage

```
cor.mlogit(x)
cov.mlogit(x)
```

Arguments

x	an `mlogit` object with random parameters and `correlation=TRUE`.

Value

A numerical matrix which returns either the correlation or the covariance matrix of the random parameters.

Author(s)

Yves Croissant

Cracker *Choice of Brand for Crakers*

Description

a cross-section

number of observations : 3292

observation : individuals

country : United States

Usage

```
data(Cracker)
```

Format

A dataframe containing :

id individuals identifiers

choice one of sunshine, keebler, nabisco, private

disp.z is there a display for brand z ?

feat.z is there a newspaper feature advertisement for brand z ?

price.z price of brand z

Source

Jain, Dipak C., Naufel J. Vilcassim and Pradeep K. Chintagunta (1994) "A random–coefficients logit brand–choice model applied to panel data", *Journal of Business and Economics Statistics*, **12(3)**, 317.

Paap, R. and Philip Hans Frances (2000) "A dynamic multinomial probit model for brand choices with different short–run effects of marketing mix variables", *Journal of Applied Econometrics*, **15(6)**, 717–744.

References

Journal of Business Economics and Statistics web site : http://www.amstat.org/publications/jbes/.

`distribution` *Functions used to describe the characteristics of estimated random parameters*

Description

`rpar` objects contain all the relevant information about the distribution of random parameters. These functions enables to obtain easily descriptive statistics, density, probability and quantiles of the distribution.

Usage

```
med(x, ...)
stdev(x, ...)
rg(x, ...)
qrpar(x, ...)
prpar(x, ...)
drpar(x, ...)
## S3 method for class 'rpar'
mean(x, norm = NULL, ...)
## S3 method for class 'rpar'
med(x, norm = NULL, ...)
## S3 method for class 'rpar'
```

```
stdev(x, norm = NULL, ...)
## S3 method for class 'rpar'
rg(x, norm = NULL, ...)
## S3 method for class 'mlogit'
mean(x, par = NULL, norm = NULL, ...)
## S3 method for class 'mlogit'
med(x, par = NULL, norm = NULL, ...)
## S3 method for class 'mlogit'
stdev(x, par = NULL, norm = NULL, ...)
## S3 method for class 'mlogit'
rg(x, par = NULL, norm = NULL, ...)
## S3 method for class 'rpar'
stdev(x, norm = NULL, ...)
## S3 method for class 'rpar'
qrpar(x, norm = NULL, ...)
## S3 method for class 'rpar'
prpar(x, norm = NULL, ...)
## S3 method for class 'rpar'
drpar(x, norm = NULL, ...)
## S3 method for class 'mlogit'
qrpar(x, par = 1, y = NULL, norm = NULL, ...)
## S3 method for class 'mlogit'
prpar(x, par = 1, y = NULL, norm = NULL, ...)
## S3 method for class 'mlogit'
drpar(x, par = 1, y = NULL, norm = NULL, ...)
```

Arguments

x	a mlogit or a rpar object,
norm	the variable used for normalization if any : for the mlogit method, this should be the name of the parameter, for the rpar method the absolute value of the parameter,
par	the required parameter(s) for the mlogit methods (either the name or the position of the parameter(s). If NULL, all the random parameters are used.
y	values for which the function has to be evaluated,
...	further arguments.

Details

mean, med, stdev and rg compute respectively the mean, the median, the standard deviation and the range of the random parameter. qrpar, prpar, drpar return functions that compute the quantiles, the probability and the density of the random parameters (note that sd and range are not generic function in R and that median is, but without ...).

Value

a numeric vector for qrpar, drpar and prpar, a numeric vector for mean, stdev and med and a numeric matrix for rg.

Author(s)

Yves Croissant

See Also

`mlogit` for the estimation of random parameters logit models and `rpar` for the description of `rpar` objects.

`effects.mlogit` *Marginal effects of the covariates*

Description

The `effects` method for `mlogit` objects computes the marginal effects of the selected covariate on the probabilities of choosing the alternatives

Usage

```
## S3 method for class 'mlogit'
effects(object, covariate = NULL,
                        type = c("aa", "ar", "rr", "ra"), data = NULL, ...)
```

Arguments

`object`	a `mlogit` object,
`covariate`	the name of the covariate for which the effect should be computed,
`type`	the effect is a ratio of two marginal variations of the probability and of the covariate ; these variations can be absolute `"a"` or relative `"r"`. This argument is a string that contains two letters, the first refers to the probability, the second to the covariate,
`data`	a data.frame containing the values for which the effects should be calculated. The number of lines of this data.frame should be equal to the number of alternatives,
`...`	further arguments.

Value

If the covariate is alternative specific, a J times J matrix is returned, J being the number of alternatives. Each line contains the marginal effects of the covariate of one alternative on the probability to choose any alternative. If the covariate is individual specific, a vector of length J is returned.

Author(s)

Yves Croissant

See Also

mlogit for the estimation of multinomial logit models.

Examples

```
data("Fishing", package = "mlogit")
Fish <- mlogit.data(Fishing, varying = c(2:9), shape = "wide", choice = "mode")
m <- mlogit(mode ~ price | income | catch, data = Fish)
# compute a data.frame containing the mean value of the covariates in
# the sample
z <- with(Fish, data.frame(price = tapply(price, index(m)$alt, mean),
                           catch = tapply(catch, index(m)$alt, mean),
                           income = mean(income)))
# compute the marginal effects (the second one is an elasticity
effects(m, covariate = "income", data = z)
effects(m, covariate = "price", type = "rr", data = z)
effects(m, covariate = "catch", type = "ar", data = z)
```

Electricity *Stated preference data for the choice of electricity suppliers*

Description

panel data

number of observations : 4308

observation : households

country : United States

Usage

```
data(Electricity)
```

Format

A dataframe containing :

choice the choice of the individual, one of 1, 2, 3, 4,

id the individual index,

pfi fixed price at a stated cents per kWh, with the price varying over suppliers and experiments, for scenario i=(1, 2, 3, 4),

cli the length of contract that the supplier offered, in years (such as 1 year or 5 years.) During this contract period, the supplier guaranteed the prices and the buyer would have to pay a penalty if he/she switched to another supplier. The supplier could offer no contract in which case either side could stop the agreement at any time. This is recorded as a contract length of 0

loci is the supplier a local company,

wki is the supplier a well-known company

todi a time-of-day rate under which the price is 11 cents per kWh from 8am to 8pm and 5 cents per kWh from 8pm to 8am. These TOD prices did not vary over suppliers or experiments: whenever the supplier was said to offer TOD, the prices were stated as above.

seasi a seasonal rate under which the price is 10 cents per kWh in the summer, 8 cents per kWh in the winter, and 6 cents per kWh in the spring and fall. Like TOD rates, these prices did not vary. Note that the price is for the electricity only, not transmission and distribution, which is supplied by the local regulated utility.

Source

Hubert J, Train K (2001) "On the similarity of classical and Bayesian estimates of individual mean pathworths", *Marketing Letters*, 12, 259-269.

Revelt D, Train K (2000) "Customer-specific taste parameters and mixed logit", Working Paper no. E00-274, Department of Economics, University of California, Berkeley.

References

Kenneth Train's home page : http://elsa.berkeley.edu/~train/.

Examples *Examples of mixed logit and multinomial probit models*

Description

This file contains several fitted mixed logit and multinomial probit models. These examples are used in the vignettes and in the man page and are stored in this file because they are pretty long to compute

Usage

```
data(Examples)
```

Format

These examples are obtained using the "Electricity" and the "Train", "Mode" and "Fishing" data set.

`Fishing` *Choice of Fishing Mode*

Description

a cross-section

number of observations : 1182

observation : individuals

country : United States

Usage

```
data(Fishing)
```

Format

A dataframe containing :

mode recreation mode choice, one of : beach, pier, boat and charter

price.beach price for beach mode

price.pier price for pier mode

price.boat price for private boat mode

price.charter price for charter boat mode

catch.beach catch rate for beach mode

catch.pier catch rate for pier mode

catch.boat catch rate for private boat mode

catch.charter catch rate for charter boat mode

income monthly income

Source

Herriges, J. A. and C. L. Kling (1999) "Nonlinear Income Effects in Random Utility Models", *Review of Economics and Statistics*, **81**, 62-72.

References

Cameron, A.C. and P.K. Trivedi (2005) *Microeconometrics : methods and applications*, Cambridge, pp. 463–466, 486 and 491–495.

Game *Ranked data for gaming platforms*

Description

a cross-section

number of observations : 91

observation : individuals

country : Netherlands

Usage

```
data(Game)
```

Format

A dataframe containing :

ch.Platform where `platform` is one of `Xbox`, `PlayStation`, `PSPortable`, `GameCube`, `GameBoy` and `PC`. This variables contain the ranking of the platforms from 1 to 6,

own.Platform these 6 variables are dummies which indicate whether the given plaform is already owned by the respondent,

age the age of the respondent,

hours hours per week spent on gaming.

Details

The data are also provided in long format (use in this case `data(Game2)`). In this case, the alternative and the choice situation are respectively indicated in the `platform` and `chid` variables.

Source

Denis Fok, Richard Paap, and Bram van Dijk (2010) "A Rank-Ordered Logit Model with Unobserved Heterogeneity in Ranking Capabilities", Journal of Applied Econometrics, forthcoming

References

Journal of Applied Econometrics data archive : http://jae.wiley.com/jae/.

HC — *Heating and Cooling System Choice in Newly Built Houses in California*

Description

a cross-section

number of observations : 250

observation : households

country : California

Usage

```
data(HC)
```

Format

A dataframe containing :

depvar heating system, one of gcc (gas central heat with cooling), ecc (electric central resistence heat with cooling), erc (electric room resistence heat with cooling), hpc (electric heat pump which provides cooling also), gc (gas central heat without cooling, ec (electric central resistence heat without cooling), er (electric room resistence heat without cooling)

ich.z installation cost of the heating portion of the system

icca installation cost for cooling

och.z operating cost for the heating portion of the system

occa operating cost for cooling

income annual income of the household

References

Kenneth Train's home page : http://elsa.berkeley.edu/~train/.

Heating — *Heating System Choice in California Houses*

Description

a cross-section

number of observations : 900

observation : households

country : California

Usage

```
data(Heating)
```

Format

A dataframe containing :

idcase id

depvar heating system, one of gc (gas central), gr (gas room), ec (electric central), er (electric room), hp (heat pump)

ic.z installation cost for heating system z (defined for the 5 heating systems)

oc.z annual operating cost for heating system z (defined for the 5 heating systems)

pb.z ratio oc.z/ic.z

income annual income of the household

agehed age of the household head

rooms numbers of rooms in the house

References

Kenneth Train's home page : http://elsa.berkeley.edu/~train/.

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hmftest *Hausman-McFadden Test*

Description

Test the IIA hypothesis (independence of irrelevant alternatives) for a multinomial logit model.

Usage

```
hmftest(x, ...)
## S3 method for class 'mlogit'
hmftest(x, z, ...)
## S3 method for class 'formula'
hmftest(x, alt.subset, ...)
```

Arguments

x	an object of class mlogit or a formula,
z	an object of class mlogit or a subset of alternatives for the mlogit method. This should be the same model as x estimated on a subset of alternatives,
alt.subset	a subset of alternatives,
...	further arguments passed to mlogit for the formula method.

Details

This is an implementation of the Hausman's consistency test for multinomial logit models. If the independance of irrelevant alternatives applies, the probability ratio of every two alternatives depends only on the characteristics of these alternatives. Consequentely, the results obtained on the estimation with all the alternatives or only on a subset of them are consistent, but more efficient in the first case. On the contrary, only the results obtained from the estimation on a relevant subset are consistent. To compute this test, one needs a model estimated with all the alternatives and one model estimated on a subset of alternatives. This can be done by providing two objects of class `mlogit`, one object of class `mlogit` and a character vector indicating the subset of alternatives, or a formula and a subset of alternatives.

Value

an object of class `"htest"`.

Author(s)

Yves Croissant

References

Hausman, J.A. and D. McFadden (1984), A Specification Test for the Multinomial Logit Model, *Econometrica*, **52**, pp.1219–1240.

Examples

```
## from Greene's Econometric Analysis p. 731

data("TravelMode",package="AER")
TravelMode <- mlogit.data(TravelMode,choice="choice",shape="long",
                          alt.var="mode",chid.var="individual")

## Create a variable of income only for the air mode

TravelMode$avinc <- with(TravelMode,(mode=='air')*income)

## Estimate the model on all alternatives, with car as the base level
## like in Greene's book.

#x <- mlogit(choice~wait+gcost+avinc,TravelMode,reflevel="car")
x <- mlogit(choice~wait+gcost+avinc,TravelMode)

## Estimate the same model for ground modes only (the variable avinc
## must be dropped because it is 0 for every observation

g <- mlogit(choice~wait+gcost,TravelMode,reflevel="car",
            alt.subset=c("car","bus","train"))

## Compute the test
```

```
hmftest(x,g)
```

Ketchup *Choice of Brand for Ketchup*

Description

a cross-section

number of observations : 4956

observation : individuals

country : United States

Usage

```
data(Ketchup)
```

Format

A dataframe containing :

hid individuals identifiers

id purchase identifiers

choice one of heinz, hunts, delmonte, stb (store brand)

price.z price of brand z

Source

Kim, Byong–Do, Robert C. Blattberg and Peter E. Rossi (1995) "Modeling the distribution of price sensitivity and implications for optimal retail pricing", *Journal of Business Economics and Statistics*, **13(3)**, 291.

References

Journal of Business Economics and Statistics web site : http://www.amstat.org/publications/jbes/.

mFormula *Model formula for logit models*

Description

Two kinds of variables are used in logit models: alternative specific and individual specific variables. mFormula provides a relevant class to deal with this specificity and suitable methods to extract the elements of the model.

Usage

```
mFormula(object)
## S3 method for class 'formula'
mFormula(object)
is.mFormula(object)
## S3 method for class 'mFormula'
model.matrix(object, data, ...)
## S3 method for class 'mFormula'
model.frame(formula, data, ..., lhs = NULL, rhs = NULL)
```

Arguments

object	for the mFormula function, a formula, for the update and model.matrix methods, a mFormula object,
formula	a mFormula object,
data	a data.frame,
lhs	see Formula
rhs	see Formula
...	further arguments.

Details

Let J being the number of alternatives. The formula may include alternative-specific and individual specific variables. For the latter, J-1 coefficients are estimated for each variable. For the former, only one (generic) coefficient or J different coefficient may be estimated.

A mFormula is a formula for which the right hand side may contain three parts: the first one contains the alternative specific variables with generic coefficient, *i.e.* a unique coefficient for all the alternatives ; the second one contains the individual specific variables for which one coefficient is estimated for all the alternatives except one of them ; the third one contains the alternative specific variables with alternative specific coefficients. The different parts are separeted by a "|" sign. If a standard formula is writen, it is assumed that there are only alternative specific variables with generic coefficients.

The intercept is necessarely alternative specific (a generic intercept is not identified because only utility differences are relevant). Therefore, it deals with the second part of the formula. As it is usual in R, the default behaviour is to include an intercept. A model without an intercept (which is

hardly meaningfull) may be specified by includint `+0` or `-1` in the second rhs part of the formula. `+0` or `-1` in the first and in the third part of the formula are simply ignored.

Specific methods are provided to build correctly the model matrix and to update the formula. The `mFormula` function is not intended to be use directly. While using the `mlogit` function, the first argument is automaticaly coerced to a `mFormula` object.

Value

an object of class `mFormula`.

Author(s)

Yves Croissant

Examples

```
data("Fishing", package = "mlogit")
Fish <- mlogit.data(Fishing, varying = c(2:9), shape = "wide", choice =
"mode")

# a formula with to alternative specific variables (price and catch) and
# an intercept
f1 <- mFormula(mode ~ price + catch)
head(model.matrix(f1, Fish), 2)

# same, with an individual specific variable (income)
f2 <- mFormula(mode ~ price + catch | income)
head(model.matrix(f2, Fish), 2)

# same, without an intercept
f3 <- mFormula(mode ~ price + catch | income + 0)
head(model.matrix(f3, Fish), 2)

# same as f2, but now, coefficients of catch are alternative specific
f4 <- mFormula(mode ~ price | income | catch)
head(model.matrix(f4, Fish), 2)
```

---

mlogit *Multinomial logit model*

Description

Estimation by maximum likelihood of the multinomial logit model, with alternative-specific and/or individual specific variables.

Usage

```
mlogit(formula, data, subset, weights, na.action, start = NULL,
       alt.subset = NULL, reflevel = NULL,
       nests = NULL, un.nest.el = FALSE, unscaled = FALSE,
       heterosc = FALSE, rpar = NULL, probit = FALSE,
       R = 40, correlation = FALSE, halton = NULL,
       random.nb = NULL, panel = FALSE, estimate = TRUE,
       seed = 10, ...)
## S3 method for class 'mlogit'
print(x, digits = max(3, getOption("digits") - 2),
    width = getOption("width"), ...)
## S3 method for class 'mlogit'
summary(object, ...)
## S3 method for class 'summary.mlogit'
print(x, digits = max(3, getOption("digits") - 2),
    width = getOption("width"), ...)
## S3 method for class 'mlogit'
print(x, digits = max(3, getOption("digits") - 2),
    width = getOption("width"), ...)
## S3 method for class 'mlogit'
logLik(object, ...)
## S3 method for class 'mlogit'
residuals(object, outcome = TRUE, ...)
## S3 method for class 'mlogit'
fitted(object, outcome = TRUE, ...)
## S3 method for class 'mlogit'
predict(object, newdata, returnData = FALSE, ...)
## S3 method for class 'mlogit'
df.residual(object, ...)
## S3 method for class 'mlogit'
terms(x, ...)
## S3 method for class 'mlogit'
model.matrix(object, ...)
## S3 method for class 'mlogit'
update(object, new, ...)
```

Arguments

x, object	an object of class mlogit
formula	a symbolic description of the model to be estimated,
new	an updated formula for the update method,
newdata	a data.frame for the predict method,
returnData	if TRUE, the data is returned as an attribute,
data	the data: an mlogit.data object or an ordinary data.frame,
subset	an optional vector specifying a subset of observations,
weights	an optional vector of weights,

na.action	a function which indicates what should happen when the data contains 'NA's,
start	a vector of starting values,
alt.subset	a vector of character strings containing the subset of alternative on which the model should be estimated,
reflevel	the base alternative (the one for which the coefficients of individual-specific variables are normalized to 0),
nests	a named list of characters vectors, each names being a nest, the corresponding vector being the set of alternatives that belong to this nest,
un.nest.el	a boolean, if TRUE, the hypothesis of unique elasticity is imposed for nested logit models,
unscaled	a boolean, if TRUE, the unscaled version of the nested logit model is estimated,
heterosc	a boolean, if TRUE, the heteroscedastic logit model is estimated,
rpar	a named vector whose names are the random parameters and values the distribution : 'n' for normal, 'l' for log-normal, 't' for truncated normal, 'u' for uniform,
probit	if TRUE, a multinomial porbit model is estimated,
R	the number of function evaluation for the gaussian quadrature method used if heterosc=TRUE, the number of draws of pseudo-random numbers if rpar is not NULL,
correlation	only relevant if rpar is not NULL, if true, the correlation between random parameters is taken into account,
halton	only relevant if rpar is not NULL, if not NULL, halton sequence is used instead of pseudo-random numbers. If halton=NA, some default values are used for the prime of the sequence (actually, the primes are used in order) and for the number of elements droped. Otherwise, halton should be a list with elements prime (the primes used) and drop (the number of elements droped).
random.nb	only relevant if rpar is not NULL, a user-supplied matrix of random,
panel	only relevant if rpar is not NULL and if the data are repeated observations of the same unit ; if TRUE, the mixed-logit model is estimated using panel techniques,
estimate	a boolean indicating whether the model should be estimated or not: if not, the model.frame is returned,
seed	,
digits	the number of digits,
width	the width of the printing,
outcome	a boolean which indicates, for the fitted and the residuals methods whether a matrix (for each choice, one value for each alternative) or a vector (for each choice, only a value for the alternative chosen) should be returned,
...	further arguments passed to mlogit.data or mlogit.optim.

Details

For how to use the formula argument, see mFormula.

The data argument may be an ordinary data.frame. In this case, some supplementary arguments should be provided and are passed to mlogit.data. Note that it is not necessary to indicate the choice argument as it is deduced from the formula.

The model is estimated using the mlogit.optim function.

The basic multinomial logit model and three important extentions of this model may be estimated.

If heterosc=TRUE, the heteroscedastic logit model is estimated. J-1 extra coefficients are estimated that represent the scale parameter for J-1 alternatives, the scale parameter for the reference alternative being normalized to 1. The probabilities don't have a closed form, they are estimated using a gaussian quadrature method.

If nests is not NULL, the nested logit model is estimated.

If rpar is not NULL, the random parameter model is estimated. The probabilities are approximated using simulations with R draws and halton sequences are used if halton is not NULL. Pseudo-random numbers are drawns from a standard normal and the relevant transformations are performed to obtain numbers drawns from a normal, log-normal, censored-normal or uniform distribution. If correlation=TRUE, the correlation between the random parameters are taken into account by estimating the components of the cholesky decomposition of the covariance matrix. With G random parameters, without correlation G standard deviations are estimated, with correlation G * (G + 1) /2 coefficients are estimated.

Value

An object of class "mlogit", a list with elements:

coefficients	the named vector of coefficients,
logLik	the value of the log-likelihood,
hessian	the hessian of the log-likelihood at convergence,
gradient	the gradient of the log-likelihood at convergence,
call	the matched call,
est.stat	some information about the estimation (time used, optimisation method),
freq	the frequency of choice,
residuals	the residuals,
fitted.values	the fitted values,
formula	the formula (a mFormula object),
expanded.formula	the formula (a formula object),
model	the model frame used,
index	the index of the choice and of the alternatives.

Author(s)

Yves Croissant

References

McFadden, D. (1973) Conditional Logit Analysis of Qualitative Choice Behavior, in P. Zarembka ed., *Frontiers in Econometrics*, New-York: Academic Press.

McFadden, D. (1974) "The Measurement of Urban Travel Demand", *Journal of Public Economics*, 3, pp. 303-328.

Train, K. (2004) *Discrete Choice Modelling, with Simulations*, Cambridge University Press.

See Also

`mlogit.data` to shape the data. `multinom` from package `nnet` performs the estimation of the multinomial logit model with individual specific variables. `mlogit.optim` for details about the optimization function.

Examples

```
## Cameron and Trivedi's Microeconometrics p.493 There are two
## alternative specific variables : price and catch one individual
## specific variable (income) and four fishing mode : beach, pier, boat,
## charter

data("Fishing", package = "mlogit")
Fish <- mlogit.data(Fishing, varying = c(2:9), shape = "wide", choice = "mode")

## a pure "conditional" model

summary(mlogit(mode ~ price + catch, data = Fish))

## a pure "multinomial model"

summary(mlogit(mode ~ 0 | income, data = Fish))

## which can also be estimated using multinom (package nnet)

library("nnet")
summary(multinom(mode ~ income, data = Fishing))

## a "mixed" model

m <- mlogit(mode ~ price+ catch | income, data = Fish)
summary(m)

## same model with charter as the reference level

m <- mlogit(mode ~ price+ catch | income, data = Fish, reflevel = "charter")

## same model with a subset of alternatives : charter, pier, beach

m <- mlogit(mode ~ price+ catch | income, data = Fish,
            alt.subset = c("charter", "pier", "beach"))
```

```
## model on unbalanced data i.e. for some observations, some
## alternatives are missing

# a data.frame in wide format with two missing prices
Fishing2 <- Fishing
Fishing2[1, "price.pier"] <- Fishing2[3, "price.beach"] <- NA
mlogit(mode~price+catch|income, Fishing2, shape="wide", choice="mode", varying = 2:9)

# a data.frame in long format with three missing lines
data("TravelMode", package = "AER")
Tr2 <- TravelMode[-c(2, 7, 9),]
mlogit(choice~wait+gcost|income+size, Tr2, shape = "long",
       chid.var = "individual", alt.var="mode", choice = "choice")

## An heteroscedastic logit model

data("TravelMode", package = "AER")
hl <- mlogit(choice ~ wait + travel + vcost, TravelMode,
             shape = "long", chid.var = "individual", alt.var = "mode",
             method = "bfgs", heterosc = TRUE, tol = 10)

## A nested logit model

TravelMode$avincome <- with(TravelMode, income * (mode == "air"))
TravelMode$time <- with(TravelMode, travel + wait)/60
TravelMode$timeair <- with(TravelMode, time * I(mode == "air"))
TravelMode$income <- with(TravelMode, income / 10)

# Hensher and Greene (2002), table 1 p.8-9 model 5
TravelMode$incomeother <- with(TravelMode, ifelse(mode %in% c('air', 'car'), income, 0))
nl <- mlogit(choice~gcost+wait+incomeother, TravelMode,
             shape='long', alt.var='mode',
             nests=list(public=c('train', 'bus'), other=c('car','air')))

# same with a comon nest elasticity (model 1)
nl2 <- update(nl, un.nest.el = TRUE)

## a probit model
## Not run:
pr <- mlogit(choice ~ wait + travel + vcost, TravelMode,
             shape = "long", chid.var = "individual", alt.var = "mode",
             probit = TRUE)

## End(Not run)


## a mixed logit model
## Not run:
rpl <- mlogit(mode ~ price+ catch | income, Fishing, varying = 2:9,
              shape = 'wide', rpar = c(price= 'n', catch = 'n'),
              correlation = TRUE, halton = NA,
              R = 10, tol = 10, print.level = 0)
summary(rpl)
```

```
rpar(rpl)
cor.mlogit(rpl)
cov.mlogit(rpl)
rpar(rpl, "catch")
summary(rpar(rpl, "catch"))

## End(Not run)

# a ranked ordered model
data("Game", package = "mlogit")
g <- mlogit(ch~own|hours, Game, choice='ch', varying = 1:12,
            ranked=TRUE, shape="wide", reflevel="PC")
```

`mlogit.data` *data.frame for logit model*

Description

shape a `data.frame` in a suitable form for the use of the `mlogit` function.

Usage

```
mlogit.data(data, choice, shape = c("wide","long"), varying = NULL,
            sep=".",alt.var = NULL, chid.var = NULL, alt.levels = NULL,
            id.var = NULL, opposite = NULL, drop.index = FALSE,
            ranked = FALSE, ...)
## S3 method for class 'mlogit.data'
mean(x, ...)
## S3 method for class 'pseries'
print(x, ...)
```

Arguments

`data`	a `data.frame`,
`x`	a `mlogit.data` or a `pseries` object,
`choice`	the variable indicating the choice made: it can be either a logical vector, a numerical vector with 0 where the alternative is not chosen, a factor with level 'yes' when the alternative is chosen
`shape`	the shape of the `data.frame`: whether `long` if each row is an alternative or `wide` if each row is an observation,
`varying`	the indexes of the variables that are alternative specific,
`sep`	the seperator of the variable name and the alternative name (only relevant for a `wide` `data.frame`),

alt.var	the name of the variable that contains the alternative index (for a long data.frame only) or the name under which the alternative index will be stored (the default name is alt),
chid.var	the name of the variable that contains the choice index or the name under which the choice index will be stored,
alt.levels	the name of the alternatives: if null, for a wide data.frame, they are guessed from the variable names and the choice variable (both should be the same), for a long data.frame, they are guessed from the alt.var argument,
id.var	the name of the variable that contains the individual index if any,
opposite	returns the opposite of the specified variables,
drop.index	should the index variables be dropped from the data.frame,
ranked	a logical value which is true if the response is a rank.
...	further arguments passed to reshape.

Value

A mlogit.data object, which is a data.frame in long format, *i.e.* one line for each alternative. It has a index attribute, which is a data.frame that contains the index of the choice made ('chid'), the index of the alternative ('alt') and, if any, the index of the individual ('id'). The choice variable is a boolean which indicates the choice made. This function use reshape if the data.frame is in wide format.

Author(s)

Yves Croissant

See Also

reshape

Examples

```
# ModeChoice is a long data.frame

data("TravelMode", package = "AER")
TM <- mlogit.data(TravelMode, choice = "choice", shape = "long",
                  alt.levels = c("air", "train", "bus", "car"))

# Same but the alt variable called mode is provided

TM <- mlogit.data(TravelMode ,choice = "choice", shape = "long",
                   alt.var = "mode")

# Same but the chid variable called individual is provided

TM <- mlogit.data(TravelMode, choice = "choice",
                   shape = "long", id.var = "individual",
                   alt.levels = c("air", "train", "bus", "car"))
```

```
# Same but with two own provided variables

TM <- mlogit.data(TravelMode, choice = "choice", shape = "long",
                  id.var = "individual", alt.var = "mode")

# Same but with two own provided variables which are deleted from the
  data.frame

TM <- mlogit.data(TravelMode, choice = "choice", shape = "long",
                  id.var = "individual", alt.var = "mode", drop.index = TRUE)


# Train is a wide data.frame with columns 'choiceid' is the choice
#  index, the alternatives are named "ch1" and "ch2", the opposite of
#  the variables is returned

data("Train", package = "mlogit")
Train <- mlogit.data(Train, choice = "choice", shape = "wide",
                     varying = 4:11, alt.levels = c("ch1", "ch2"), sep = "",
                     opposite = c("price", "time", "change", "comfort"))


# Car is a wide data.frame

data("Car", package = "mlogit")
Car <- mlogit.data(Car, varying = 5:70, shape = "wide", sep = "",
                   choice = "choice", alt.levels = 1:6)

data("HC", package = "mlogit")
HC <- mlogit.data(HC, choice = "depvar", varying=c(2:8, 10:16), shape="wide")

# Game is a data.frame in wide format for which the response is a
#  ranking variable

data("Game", package = "mlogit")
G <- mlogit.data(Game, shape="wide", varying = 1:12, alt.var = 'platform',
                  drop.index = TRUE, choice="ch", ranked =TRUE)

# Game2 contains the same data, but in long format
data("Game2", package = "mlogit")
G2 <- mlogit.data(Game2,  shape='long', choice="ch", alt.var = 'platform', ranked = TRUE)
```

`mlogit.optim` *Non-linear minimization routine*

Description

This function performs efficiently the optimization of the likelihood functions for multinomial logit models

Usage

```
mlogit.optim(logLik, start, method = c("bfgs", "nr", "bhhh"), iterlim = 2000,
             tol = 1E-06, ftol = 1e-08, steptol = 1e-10,
             print.level = 0, constPar = NULL, ...)
```

Arguments

logLik	the likelihood function to be maximized,
start	the initial value of the vector of coefficients,
method	the method used, one of 'nr' for Newton-Ralphson, 'bhhh' for Berndt-Hausman-Hall-Hall and 'bfgs',
iterlim	the maximum number of iterations,
tol	the value of the criteria for the gradient,
ftol	the value of the criteria for the function,
steptol	the value of the criteria for the step,
print.level	one of (0, 1, 2), the details of the printing messages. If 'print.level=0', no information about the optimization process is provided, if 'print.level=1' the value of the likelihood, the step and the stoping criteria is printing, if 'print.level=2' the vectors of the parameters and the gradient are also printed.
constPar	a numeric or a character vector which indicates that some parameters should be treated as constant,
...	further arguments passed to f.

Details

The optimization is performed by updating, at each iteration, the vector of parameters by the amount step * direction, where step is a positive scalar and direction = H^-1 * g, where g is the gradient and H^-1 is an estimation of the inverse of the hessian. The choice of H^-1 depends on the method chosen :

if method='nr', H is the hessian (i.e. is the second derivates matrix of the likelihood function),

if method = 'bhhh', H is the outer-product of the individual contributions of each individual to the gradient,

if method = 'bfgs', H^-1 is updated at each iteration using a formula that uses the variations of the vector of parameters and the gradient. The initial value of the matrix is the inverse of the outer-product of the gradient (i.e. the bhh estimator of the hessian).

The initial step is 1 and, if the new value of the function is less than the previous value, it is divided by two, until a higher value is obtained.

The routine stops when the gradient is sufficiently close to 0. The criteria is g * H^-1 * g which is compared to the tol argument. It also may stops if the number of iterations equals iterlim.

The function f has a initial.value argument which is the initial value of the likelihood. The function is then evaluated a first time with a step equals to one. If the value is lower than the initial value, the step is divided by two until the likelihood increases. The gradient is then computed and the function returns as attributes the gradient is the step. This method is more efficient than other functions available for R :

For the optim and the maxLik functions, the function and the gradient should be provided as separate functions. But, for multinomial logit models, both depends on the probabilities which are the most time-consuming elements of the model to compute.

For the nlm function, the fonction returns the gradient as an attribute. The gradient is therefore computed at each iteration, even when the function is computed with a step that is unable to increase the value of the likelihood.

Previous versions of mlogit depended on the 'maxLik' package. We kept the same interface, namely the start, method, iterlim, tol, print.level and constPar arguments.

The default method is 'bfgs', which is known to perform well, even if the likelihood function is not well behaved and the default value for print.level=1, which means moderate printing.

A special default behavior is performed if a simple multinomial logit model is estimated. Indeed, for this model, the likelihood function is concave, the analytical hessian is simple to write and the optimization is straightforward. Therefore, in this case, the default method is 'nr' and print.level=0.

Value

a list that contains the followings elements :

optimum	the value of the function at the optimum, with attributes: gradi a matrix that contains the contribution of each individual to the gradient, gradient the gradient and, if method='nr' hessian the hessian,
coefficients	the vector of the parameters at the optimum,
est.stat	a list that contains some information about the optimization : 'nb.iter' the number of iterations, 'eps' the value of the stoping criteria, 'method' the method of optimization method used, 'message'

Author(s)

Yves Croissant

MobilePhones *Stated Preferences survey for mobile phones*

Description

a cross-section from 2003

number of observations : 11184

observation : individuals

country : Netherland

Usage

```
data(MobilePhones)
```

Format

A dataframe containing :

alt the alternative, denoted by 1 or 2

choice 1 if the alternative is chosen, 0 otherwise

price purchase price in euros (100, 135 or 170)

mincost cost of a minute in euros (0.25, 0.30 or 0.35)

extras extra features of the telephone : a factor with levels `games`, `internet` (which means games and internet) and `camera` (which means games and internet and camera)

network a factor with levels `KPNVodaphone` and `other`

sms the cost of an sms in euros (0.17 or 0.23)

design a factor with levels `basic` or `trendy`

Source

Sandor Zsolt and Philip Hans Franses (2009) "Consumer price evaluations through choice experiments", *Journal of Applied Econometrics*, **24**, 517–535.

References

Journal of Applied Econometrics data archive : http://jae.wiley.com/jae/.

`Mode` *Mode Choice*

Description

a cross-section

number of observations : 453

observation : individuals

Usage

```
data(Mode)
```

Format

A dataframe containing :

choice one of car, carpool, bus or rail

cost.z cost of mode z

time.z time of mode z

References

Kenneth Train's home page : http://elsa.berkeley.edu/~train/.

ModeCanada *Mode Choice for the Montreal-Toronto Corridor*

Description

a cross-section

number of observations : 3880

observation : individuals

Usage

```
data(ModeCanada)
```

Format

A dataframe containing :

case the individual index

alt the alternative, one of train, car, bus and air,

choice one if the mode is chosen, zero otherwise,

cost monetary cost,

ivt in vehicule time,

ovt out vehicule time,

frequency frequency,

income income,

urban urban,

noalt the number of alternatives available.

References

Bhat, Chandra R. (1995) "A heteroscedastic extreme value model of intercity travel mode choice", *Transportation Research Part B*, **29(6)**, 471-483.

Koppelman Franck S. and Chieh-Hua Wen (2001) "The paired combinatorial logit model:properties, estimation and application", *Transportation Research Part B*, 75-89.

Wen, Chieh-Hua and Franck S. Koppelman (2001) "The generalized nested logit model", *Transportation Research Part B*, 627-641.

Examples

```
data("ModeCanada", package = "mlogit")
bususers <- with(ModeCanada, case[choice == 1 & alt == "bus"])
ModeCanada <- subset(ModeCanada, !case %in% bususers)
ModeCanada <- subset(ModeCanada, nchoice == 4)
ModeCanada <- subset(ModeCanada, alt != "bus")
ModeCanada$alt <- ModeCanada$alt[drop = TRUE]
KoppWen00 <- mlogit.data(ModeCanada, shape='long', chid.var = 'case',
                         alt.var = 'alt', choice='choice',
                         drop.index=TRUE)
pcl <- mlogit(choice~freq+cost+ivt+ovt, KoppWen00, reflevel='car',
              nests='pcl', constPar=c('iv.train.air'))
```

`plot.mlogit` *Plot of the distribution of estimated random parameters*

Description

Methods for `rpar` and `mlogit` objects which provide a plot of the distribution of one or all of the estimated random parameters

Usage

```
## S3 method for class 'mlogit'
plot(x, par = NULL, norm = NULL,
                        type = c("density", "probability"), ...)
## S3 method for class 'rpar'
plot(x, norm = NULL, type = c("density","probability"), ...)
```

Arguments

`x`	a `mlogit` or a `rpar` object,
`type`	the function to be plotted, whether the density or the probability density function,

par	a subset of the random parameters ; if NULL, all the parameters are selected,
norm	the coefficient's name for the mlogit method or the coefficient's value for the rpar method used for normalization,
...	further arguments, passed to plot.rpar for the mlogit method and to plot for the rpar method.

Details

For the rpar method, one plot is drawn. For the mlogit method, one plot for each selected random parameter is drawn.

Author(s)

Yves Croissant

See Also

mlogit for the estimation of random parameters logit models and rpar for the description of rpar objects and distribution for functions which return informations about the distribution of random parameters.

---

rpar *random parameter objects*

Description

rpar objects contain the relevant information about estimated random parameters. The homonymous function extract on rpar object from a mlogit object.

Usage

```
rpar(x, par = NULL, norm = NULL, ...)
```

Arguments

x	a mlogit object,
par	the name or the index of the parameters to be extracted ; if NULL, all the parameters are selected,
norm	the coefficient used for normalization if any,
...	further arguments.

Details

mlogit objects contain an element called rpar which contain a list of rpar objects, one for each estimated random parameter. The print method prints the name of the distribution and the parameter, the summary behave like the one for numeric vectors.

Value

a rpar object, which contain :

dist	the name of the distribution,
mean	the first parameter of the distribution,
sigma	the second parameter of the distribution,
name	the name of the parameter,

Author(s)

Yves Croissant

See Also

mlogit for the estimation of a random parameters logit model.

---

scoretest *The three tests for mlogit models*

Description

Three tests for mlogit models: specific methods for the Wald test and the likelihood ration test and a new function for the score test

Usage

```
scoretest(object, ...)
## S3 method for class 'mlogit'
scoretest(object, ...)
## S3 method for class 'mlogit'
waldtest(object, ...)
## S3 method for class 'mlogit'
lrtest(object, ...)
```

Arguments

object	an object of class mlogit or a formula,
...	two kinds of arguments can be used. If "mlogit" arguments are introduced, initial model is updated using these arguments. If "formula" or other "mlogit" models are introduced, the standard behavior of "waldtest" and "lrtest" is followed.

Details

The "scoretest" function and "mlogit" method for "waldtest" and "lrtest" from the "lmtest" package provides the infrastructure to compute the three tests of hypothesis for "mlogit" objects.

The first argument must be a "mlogit" object. If the second one is a fitted model or a formula, the behaviour of the three functions is the one of the default methods of "waldtest" and "lrtest": the two models provided should be nested and the hypothesis tested is that the constrained model is the 'right' model.

If no second model is provided and if the model provided is the constrained model, some specific arguments of "mlogit" should be provided to descibe how the initial model should be updated. If the first model is the unconstrained model, it is tested versus the 'natural' constrained model; for example, if the model is a heteroscedastic logit model, the constrained one is the multinomial logit model.

Value

an object of class "htest".

Author(s)

Yves Croissant

Examples

```
library("mlogit")
data("TravelMode", package = "AER")
ml <- mlogit(choice ~ wait + travel + vcost, TravelMode,
             shape = "long", chid.var = "individual", alt.var = "mode")
hl <- mlogit(choice ~ wait + travel + vcost, TravelMode,
             shape = "long", chid.var = "individual", alt.var = "mode",
             method = "bfgs", heterosc = TRUE)
lrtest(ml, hl)
waldtest(hl)
scoretest(ml, heterosc = TRUE)
```

Telephone *Choice among residential telephone service options for local calling*

Description

a cross-section from 1984

number of observations : 434

observation : households

country : United-States

Usage

```
data(Train)
```

Format

A dataframe containing :

choice a logical which indicates if the alternative has been chosen

service a telephone service option

household the household index

cost the logarithm of the cost

Source

Walker J.L., Ben-Akiva M. and D. Bolduc (2007) "Identification of parameters in normal error component logit-mixture (NECLM) models", *Journal of Applied Econometrics*, **22**, 1095–1125.

Train K.E., Mc Fadden D. and M. Ben-Akiva (1987) "The demand for local telephone service: a fully discrete model of residential calling patterns and service choices", *Rand Journal of Economics*, **18(1)**, 109–123

References

Journal of Applied Econometrics data archive : http://jae.wiley.com/jae/.

TollRoad *Stated Preferences survey for a toll road*

Description

a panel from 1999-2000

number of observations : 1448 observations for 548 individuals

observation : individuals

country : United-States

Usage

```
data(TollRoad)
```

Format

A dataframe containing :

id the individual id

src the source of the data, one of `br` : revealed preference survey conducted by the Brookings Institution, `bs` : stated preferences survey conducted by the Brookings Institution and `cal` : revealed preferences survey conducted by the Californian Polytechnic State University

route the route chosen, one of `express` (the toll-road) or `freeway` (the free road)

toll.alt the monetary cost of the road (a for the free road)

time.alt the median time of the trip on both highways for the given schedule

reliability.alt the reliability of the trip length on both highways for the given schedule, measured by the difference between the 80th and the 50th percentile of the trip length

occupance the number of people in the car

size the household size

sex one of `male` or `female`

flexibility does the respondent declare having a flexible arrival time, a factor with levels `yes` or `no`

distance trip distance in miles

commute a long-commute for trips longer than 45 minutes (a factor with levels `yes` or `no`)

age3050 a factor with levels `yes` if the respondent is between 30 and 50 years old, `no` otherwise

income a factor with levels `low`, `medium` and `high`

Source

Kenneth A. Small, Clifford Winston, Jia Yan (2005) "Uncovering the distribution of motorists' preferences for travel time and reliability", *Econometrica*, **73(4)**, 1367-1382.

References

Econometrica data archive

`Train` *Stated Preferences for Train Traveling*

Description

a cross-section from 1987

number of observations : 2929

observation : individuals

country : Netherland

Usage

```
data(Train)
```

Format

A dataframe containing :

id individual identifient

choiceid choice identifient

choice one of choice1, choice2

pricez price of proposition z (z=1,2) in cents of guilders

timez travel time of proposition z (z=1,2) in minutes

comfortz comfort of proposition z (z=1,2), 0, 1 or 2 in decreasing comfort order

changez number of changes for proposition z (z=1,2)

Source

Meijer, Erik and Jan Rouwendal (2005) "Measuring welfare effects in models with random coefficients", *Journal of Applied Econometrics*, **forthcoming**.

Ben–Akiva, M., D. Bolduc and M. Bradley (1993) "Estimation of travel choice models with randomly distributed values of time", *Transportation Research Record*, **1413**, 88–97.

Carson, R.T., L. Wilks and D. Imber (1994) "Valuing the preservation of Australia's Kakadu conservation zone", *Oxford Economic Papers*, **46**, 727–749.

References

Journal of Applied Econometrics data archive : http://jae.wiley.com/jae/.

`Tuna` *Choice of Brand for Tuna*

Description

a cross-section

number of observations : 13705

observation : individuals

country : United States

Usage

```
data(Tuna)
```

Format

A dataframe containing :

hid individuals identifiers

id purchase identifiers

choice one of skw (Starkist water), cosw (Chicken of the sea water), pw (store–specific private label water), sko (Starkist oil), coso (Chicken of the sea oil)

price.z price of brand z

Source

Kim, Byong–Do, Robert C. Blattberg and Peter E. Rossi (1995) "Modeling the distribution of price sensitivity and implications for optimal retail pricing", *Journal of Business Economics and Statistics*, **13(3)**, 291.

References

Journal of Business Economics and Statistics web site : http://www.amstat.org/publications/jbes/.

`vcov.mlogit` *vcov method for mlogit objects*

Description

The `vcov` method for `mlogit` objects extract the covariance matrix of the coefficients, the errors or the random parameters.

Usage

```
## S3 method for class 'mlogit'
vcov(object, what = c('coefficient', 'errors', 'rpar'),
                        type = c('cov', 'cor', 'sd'), reflevel = NULL, ...)
```

Arguments

`object`	a `mlogit` object,
`what`	indicates which covariance matrix has to be extracted : the default value is `coefficients`, in this case, `vcov` behaves as usual. If `what` equals `errors` the covariance matrix of the errors of the model is returned. Finally, if `what` equals `rpar`, the covariance matrix of the random parameters are extracted.
`type`	with this argument, the covariance matrix may be returned (the default) ; the correlation matrix and the standard deviation vector may also be extracted.
`reflevel`	relevent for the extraction of the errors of a multinomial probit model ; in this case the covariance matrix is of error differences is returned and, with this argument, the alternative used for differentiation is indicated.
`...`	further arguments.

Details

This new interface replaces the `cor.mlogit` and `cov.mlogit` functions which are deprecated.

Author(s)

Yves Croissant

See Also

`mlogit` for the estimation of multinomial logit models.

`Yogurt` *Choice of Brand for Yogurts*

Description

a cross-section

number of observations : 2412

observation : individuals

country : United States

Usage

```
data(Yogurt)
```

Format

A dataframe containing :

id individuals identifiers

choice one of yoplait, dannon, hiland, weight (weight watcher)

feat.z is there a newspaper feature advertisement for brand z ?

price.z price of brand z

Source

Jain, Dipak C., Naufel J. Vilcassim and Pradeep K. Chintagunta (1994) "A random–coefficients logit brand–choice model applied to panel data", *Journal of Business and Economics Statistics*, **12(3)**, 317.

References

Journal of Business Economics and Statistics web site : http://www.amstat.org/publications/jbes/.

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